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Agent-based Models and the Spatial Sciences

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Abstract

Agent-based models (ABMs) are used in the spatial sciences as building-blocks for computer simulation. ABMs have a range of advantageous attributes, not least of which is their flexibility in representing dynamic and highly adaptive physical or human phenomena. ABMs facilitate the exploration of ideas about the myriad of ways that geographical systems develop, behave, interact and evolve, often supporting experimentation with geographical systems in ways that are simply not possible in the real world. Indeed, in many cases, ABMs are developed from the bottom up, pedagogically, as a tool in building theory. Geographers' work with ABMs has helped to strengthen existing ties with related disciplines such as computer science and informatics, ecology, sustainability science, economics, anthropology, political science and the earth sciences. Primarily because of the value placed on spatial science and behavioral geography in agent-based modeling, work of this kind is helping to infuse geographical perspectives and 'spatial thinking' into these fields. This article reviews the development of agent-based modeling in the spatial sciences, its current uses and applications in physical and human geography and potential future trends in its research and development.

1 Introduction

Put briefly, agent-based models (ABMs) are tools for modeling by intelligent and proactive information processing. Agents are used as the computational 'bricks' in constructing computer simulation environments for experimentation and exploration of ideas. Although they are a fitting progression of a long-standing research agenda in behavioral geography, quantitative geography and Geographic Information Science (GIScience), ABMs represent a relatively new methodology in the spatial sciences, but use of ABMs in the spatial sciences is popular and influential and, importantly, simulation with spatial ABMs is catalysing the diffusion of 'spatial thinking' through and beyond the spatial sciences in significant ways.

The spatial sciences are an umbrella for geographic research, with a primary focus on representation of space in its diverse forms. Spatial science is usually associated with methodology to measure, analyse and represent spatial attributes of phenomena, often across a variety of substantive interests. 'Modeling' is a core component of the spatial sciences; models are used by geographers to develop and test theories, ideas and hypotheses and to convey those ideas and concepts in teaching and education. Spatial simulation (the procedure of 'using' models, for inductive or deductive reasoning, for example) is central to the analysis and interpretation of geographic data for experimentation and for extrapolation ('forecasting') about future what-if scenarios or to interpolate gaps or unknowns in our understanding of systems. Models can even be used to 'hindcast' about the origins of geographical phenomena when their evolution over space and time is less than certain. In each of these instances, the model serves as a vehicle or apparatus that allows for theory to be allied with data in some sort of analytical framework.

Originally, the analytical medium for spatial models was mathematics: the bid-rent model for factor-substitution in residential location and urban commuting (Alonso 1960), the gravity model of spatial interaction (Fotheringham and O'Kelly 1989) and early models of innovation diffusion (Hägerstrand 1967) are all classic examples that are now part of the core corpus of spatial science. Agent-based modeling represents something of a departure from traditional methods, which have often employed computers (for calculation, visualization, data storage etc.), but have not generally used computation (algorithmic functions, knowledge discovery schemes, data access structures etc.). This has changed, particularly in the last 20 years, as spatial modeling began to evolve in tandem with GIScience, bringing spatial modeling into alignment with advanced simulation techniques in computer science (Torrens 2009).

This article charts the development and application of agent-based modeling in the spatial sciences and its use in geographical inquiry outside the spatial sciences. The history of agent-based modeling in the spatial sciences is outlined in section 2 and the formulation of ABMs in simulation is discussed in section 3. Applications of those schemes to a variety of substantive phenomena are discussed in section 4 (and further details are provided in Appendices 1 and 2). The article concludes in sections 5 and 6 with a discussion of current trends in advancing spatial agent-based modeling and likely futures for the field.

2 A Brief Intellectual History of ABMs

Agent-based models are, at heart, a medium for information processing and exchange. At their cores, ABMs are automata (computable media for connecting information and processing) and they therefore share their origins with those of all digital computers, in Turing's (1936) work on the computability of mathematics and von Neumann's (1951) and Ulam's (1969) early designs for digital computers. ABMs usually manifest as an information system and in this way they are connected to information theory (Shannon 1948). Moreover, many ABMs ascribe a level of intelligence to their agents by endowing them with the ability to reason (often proactively) about the information that they process in a formal manner and ABMs may also be considered as drawing on ideas from cybernetics (Weiner 1948), and intelligent machines (Turing 1950).

Agent-like models were first introduced to the spatial sciences in the 1960s and 1970s as cellular automata (CA), which were used to model urban growth and land-use as exchanges of information between cellular representations of land units and information processing rules that mimicked general principles of urbanization and development (Chapin and Weiss 1962; Nakajima 1977; Tobler 1970). Fully-fledged agent-based modeling, in which similar capabilities were introduced without the constraint of cellular spatial data structures (Figure 1), began to appear in the spatial sciences more stealthily and at a much later time. The earliest geographical ABMs were developed by Itami et al. (1988) in the late-1980s to model recreation and wayfinding behavior along trails. ABMs also trickled into spatial science through modeling work being done in the social sciences (Epstein and Axtell 1996), particularly from model development in game theory (Schelling 1978) and 'social physics' (Helbing and Molnár 1995). At the same time, ABMs were being developed in ecology (De Angelis and Gross 1992), biology (Ermentrout and Edelstein-Keshet 1993), and entomology (Bonabeau et al. 1999). The development of spatial ABMs in this context was closely related to a growing and interdisciplinary focus on the role of space and spatial behavior in conceptualizing complex adaptive systems [see Manson (2001), Batty (2005), and Benenson and Torrens (2004) for a more complete history].

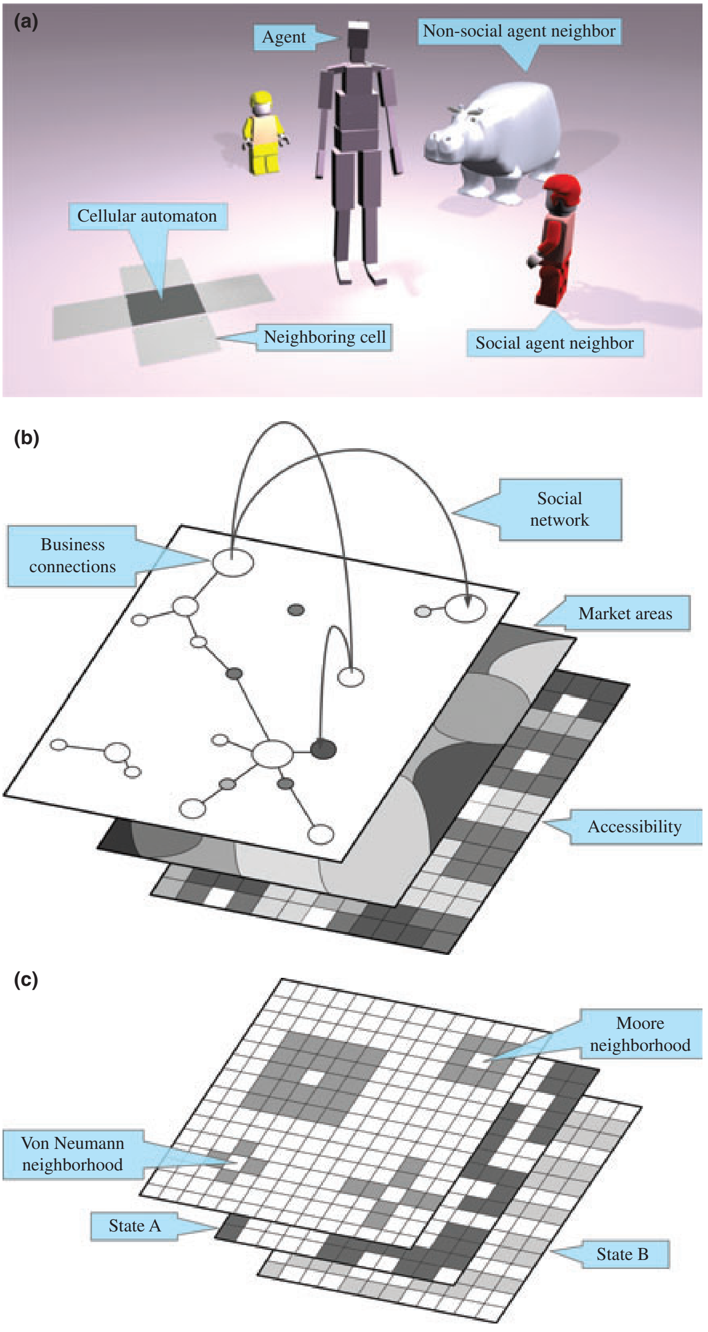

Fig. 1. (a) Equivalent spatial footprints and neighbor relationships are shown for cellular automata and an agent in an agent-based model. Cellular automata are bounded discretely by a cellular geography and neighborhood relationships are limited to adjacency between cells. By contrast, in agent-based models, agents may form neighborhood relationships with any entity in their environment. (b) Cellular automata neighborhoods are often regular and symmetrical in geometry and fixed in space and time. (c) Agent-based models may employ flexible neighborhood relationships, including relative spaces such as social networks, in addition to using cellular spaces that may denote additional state information.

3 How ABMs (and their variants) work

Agent-based modeld (ABMs) are 'models'. They are most often used as *in silico* (using simulated, computational environments or substrates) abstractions of something from the real world or something that exists in conceptual form. ABMs act as a surrogate representation for the purposes of experimentation or scenario-building. Agents can represent a variety of spatial entities and phenomena: car drivers in a traffic system (Torrens 2005), cancer cells in the body (Deutsch and Dormann 2004), or a spatial task that forms part of a cognitive action (Frank et al. 2001b), for example. Moreover, ABMs are 'agent-based'. They are a particular class of model used as proxies for agency or as vehicles to study agency. Agency is the source of great variety in the design and application of ABMs, largely because this property broadens the scope of agent-based modeling from computing to any discipline in which agency is important or even relevant. The list of related disciplines is long and includes geography (Benenson and Torrens 2004), sociology (Macy and Willer 2002), ecology (Balzter et al. 1998), anthropology (Kohler and Gumerman 2001), economics (Tesfatsion 1997), physics (Schweitzer 2003), computer graphics (Reynolds 1999), agriculture (Berger 2001), and psychology (Griffin 2005) (Appendix 1). A variety of development tools also exist for constructing ABMs (Appendix 2).

3.1 TYPES OF AGENT-BASED MODELS

Several varieties of ABM are used in spatial modeling. As mentioned already, 'cellular automata' (Batty 1997) are related to agent-automata and are distinguished by bounding agents' processing within a discrete cellular boundary that is connected to other like automata using neighborhoods of interaction (Figure 1). 'Individual-based models' (Grimm 1999) are usually designed to represent the behavior of a single agent. This might include an institution, a mind or even an organism and computation is usually focused on the agent's internal decision-making processes in these cases. 'Multi-agent models' are generally built with many individual agents, each of which may play a different role or assume a set of distinct tasks in a model of collective behavior. Such models generally focus on the interactions between individuals or units. 'Multi-agent systems' adopt a synoptic view, often from the bottom-up, to consider individual agency in the context of a larger or collaborative phenomenon. Agents in a multi-agent system usually interact with (or within) an environment that is modeled explicitly in simulation (Manson and Evans 2007). Often, the system that is considered is treated as a complex adaptive system and the interplay between agents and agent attributes is used to explore issues of emergence, feedback, self-organization, phase shift and so on (Batty 2005; Batty and Torrens 2005). 'Intelligent agents' are usually designed with a level of artificial intelligence or to explore the components of general or task-specific intellect, often through automated problem-solving. Spatial ABMs of intelligence are generally built to represent spatial thinking or cognition (Frank et al. 1992). There is also a class of agents that might best be described as *bots* (Leonard 1997), owing to the resemblance that they share with robots. Bots are designed to automate tasks in information systems and spatial bots are usually designed to retrieve and process data in a Geographic Information System (Zhang and Tsou 2009) or they are built to deploy spatial reasoning in information systems generally.

There exist several interrelationships between agent-based approaches and other formal schemes for computing, largely because agents are, fundamentally, computers. ABMs often make use of artificial intelligence (Russell and Norvig 1995) (i.e. formal schemes that use computational reasoning) to process data input to an agent. This could include machine-learning of spatial abilities (Lee et al. 2007), optimization heuristics for calculating shortest paths in path-planning (Sud et al. 2008), or pattern recognition for determining salient features (Shao and Terzopoulos 2007). Knowledge discovery and data-mining (KDD) is a method from artificial intelligence (Fayyad et al. 1996) that is closely related to agent bots. KDD is useful in agent-based modeling as a formal mechanism for proactively obtaining input for agents and for registering input to libraries of (often ontological or semantic) information. KDD is used in this way, for example, to develop agent-based web services for retrieving geographic data from spatial databases (Yu and Peuquet 2009). Distributed artificial intelligence (Ferber 1999) is concerned with communication and distributed processing among many agents and the methodology is of relevance to multi-agent systems because of the plurality of agents used in multi-agent approaches and requirements for treating agents' information exchange and related issues of information fidelity and uncertainty. Distributed artificial intelligence is therefore useful in designing agent swarms in insect models (Bonabeau 2002) or organizational interaction in models of communities (Anderies et al. 2004), for example. In a similar tradition, genetic algorithms (Mitchell 1998) are often used in agent-based modeling for expressing how information might mutate when exchanged between agents or when contextualized by a particular agent or environment (Epstein and Axtell 1996). Artificial neural networks (ANNs) are a data structure for encoding information, computationally, as a series of dynamically-weighted nodes and links. ANNs are often developed by passing information through the network using a series of training exercises that assign weights to network nodes based on a statistical fitting scheme (Gurney 1997). ABMs can be designed as ANNs by considering agents as nodes and their inter-relationships as links and such schemes have been used to build ABMs of land-use transition, in which the relative weighting for transition rules have been derived from ANN training (Li and Yeh 2002). Similar schemes employ fuzzy logic (Klir and Yuan 1994; Fonte and Lodwick 2004) in determining proportional membership of agents' state variables, when determining transition between land-use or land cover, for example (Wu 1998). ABMs are intricately bound to Geographic Information Systems (GIS) and may be built within a GIS (Wagner 1997), or they may be used to animate GIS dynamically through some sort of loose- or tight-coupling of their data models (Brown et al. 2005). Other variants of ABM actually incorporate functionality from GIScience to extend their processing (Torrens and Benenson 2005). The consociation between geographic information technologies and ABMs has led to the induction of agent-based modeling into the core of GIScience (DiBiase et al. 2006).

3.2 BUILDING AGENT-BASED MODELS

The scaffolding used in constructing ABMs is relatively straightforward, although almost limitless configuration is possible with this basic design. Many (although not necessarily all) ABMs are built as 'agent-automata', based on a state–rule–input architecture as follows:

$$A \sim \{S, R, I\} \sim \begin{cases} S = S^1, S^2, \ldots, S^k_{i,t} \\ R : \{S_t, I_t\} \rightarrow S_{t+1} \end{cases}$$

The A in the equation above represents an automaton with states $k_{it} \in S$ that describe its attributes, perhaps in a particular place (i) and/or time (t); states may be derived from a discrete set, indexed from 1 to k, for example, that might represent different 'layers' (Figure 1) of potential vegetation coverage on a landscape (Manson and Evans 2007) or the spatial factors that a mover uses to evaluate the utility of housing choices in a property submarket (Torrens 2007b), for example. The automaton's states are malleable to the influence of a rule, or set of rules $\{R\}$, that could be used to govern transition among states from $\{S\}$ as time changes from $t \rightarrow t+1$, taking the state at time t and input $\{I_{i,t}\}$ from other automata or external stimuli into consideration. Rules are generally used to represent spatial processes. Input (I) is almost always determined spatially (I_i) or spatio-temporally ($I_{i,t}$) in spatial science. Indeed, the interplay between states, rules, input, geography, and time offers the flexibility in a modeling methodology to cover 'everything' of interest to a geographer and agent-automata are universal computers (Turing 1936, 1938): anything that can be computed can be represented or evaluated by an agent automaton.

We can consider an agent's agency, computationally, by characterizing states, rules and input across any dimension of the agency that we are interested in. We could, for example, index kinetic states in $\{S\}$, where S^1 denotes mobility and S^2 denotes stationarity. The rule R in this example might be purely physical (Reynolds 1999), as would be the case if it were plucked from Newton's (1687) laws of motion, for example. Alternatively, the rule could represent a decision tree to mimic a commuters' trip-making by automobile (Nagel et al. 1998) as they balance a journey-to-work while also factoring in detours to drop children to school, deposit dry-cleaning and collect a co-worker for the car-pool. We could formulate this rule as a set of if–then conditions: 'if I am running late, then skip the trip to dry cleaners', or 'if highway traffic is congested, then take a route that uses side-streets'. We could also specify the rule mathematically using an equation, such as a discrete choice formulation that would assign relative weights to different choice-points in the decision tree (McFadden 1974). Alternatively, we might use an algorithm or heuristic, such as a shortest-path solver (Dijkstra 1959) that is designed to minimize time spent in the car and cost in terms of gasoline consumption.

Agency could be used to represent an agent's 'spatial thinking'. Consider, for example, an agent in a social crowd (Arthur 1994), using its spatial cognition to understand its surroundings: various states can be used by the agent to register conversations and body language, the spatial patterns and compositions of social groups and the geography of social events. These could be based on distance metrics (physical separation, social dissonance), adjacency (neighborliness, cardinality), spatial structure (compactness, isolation), spatial composition (texture, heterogeneity), kinetics (movement vector, angular velocity), group properties (membership, centrality) or roles (dominance, leadership), for example. These characteristics (states) could be used by the agent to classify its own conditions as well as those of the objects in its environment. Rules may be used to determine the spatial thinking or cognition that an agent employs in processing this information, e.g. by sorting, filtering, juxtaposing, measuring and so on. Agents might also use rules to mimic vision, distance and depth perception, pattern recognition, anticipation or attention. Using these schemes, an agent could, for example (Figure 2), use its spatial awareness and cognition to assess the social geography in a room and to select an appropriate conversation to join in a crowd of strangers.

4 Applications of ABMs in the Spatial Sciences (and Geographical Applications of ABMs in Other Sciences)

Agent-based models have been deployed for a wide variety of geographical applications, both within and outside the spatial sciences. Indeed, space and spatial thinking have emerged as a central concept in agent-based modeling with the result that the methodology is, to some extent, catalyzing infusion of spatial science into other disciplines,

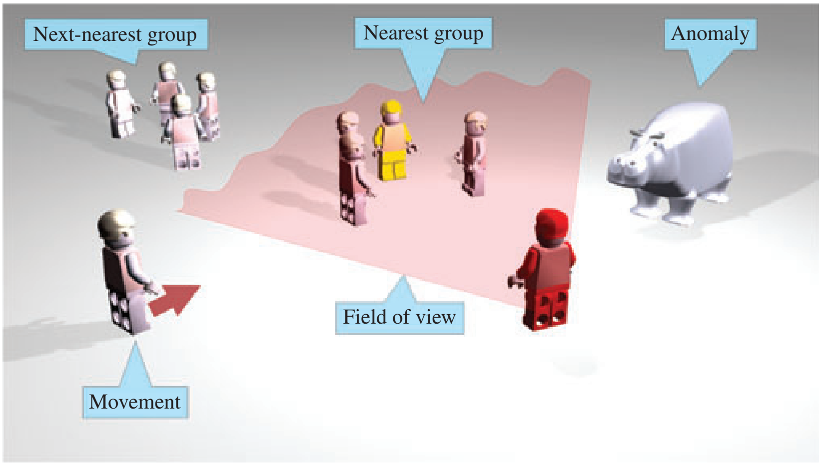

Fig. 2. An agent (red) enters a social environment and uses its spatial awareness to identify conditions around it (groups, movement, anomalies). Within its field of view, the red agent makes eye contact with the yellow agent and transitions from a state of scanning the room for conversation partners to a state of movement to engage with the yellow agent.

reversing the trend in quantitative geography, for example, for the field to adopt (or to poach) methodology from other sciences.

4.1 MOVEMENT GEOGRAPHY

Agent-based models are used to model movement at macro-, meso-, and micro-scales of study, for individuals and for groups or as flows. In physics applications, for example, agents are used to mimic particle movement in gaseous, liquid and granular flow (Henderson 1971). Similar schemes have been used to model pedestrian movement in crowd flow (Hughes 2003). In these cases, the agency responsible for movement is limited to physical forces, but in other applications the spatial cognition of agents relative to their surroundings and their own 'mental map' is modeled, allowing for time geography (Paris et al. 2007), way-finding (Sud et al. 2008), and affective movement (Pelechano and Badler 2006) to be incorporated. ABMs are also popularly used in 'micro-simulating' traffic on highways as a local-scale extension to classic four-step travel models (Hensher et al. 2004). In these cases, agents are used to represent cars and their drivers, as well as their proactive and reactive decisions to accelerate, brake and change lanes (Nagel and Schreckenberg 1992). In some examples, ABMs have been used to reproduce traffic congestion at macro-scales (an entire city), synthetically and from the bottom-up of the micro-scale (individual drivers) (Nagel et al. 1998). There is also a cohort of geographers using ABMs to model meso-scale movement of families through the urban landscape and its relationship to social anthropology; such models are most often used to simulate housing search and residential choice across the lifecycle of families (Benenson 1999; Torrens 2007b; Waddell 2000).

4.2 GROWTH MODELS

Models of urbanization and urban growth are particularly well-suited to agent-based methodologies and agents have been developed as synthetic settlers, developers and relocating households faced with location choices amid economic, physical, social, cultural and environmental geographies (Benenson and Torrens 2004; Torrens 2006). CA-based models of urbanization and urban growth have also been built with agent-based functionality, largely to simulate the dynamics of land-use transition in city-systems or to evaluate the role of complex adaptive human, social and economic dynamics in shaping the rate and pattern of urbanization under varying scenarios (Clarke et al. 2007; Engelen et al. 2002; Torrens 2006).

4.3 SOCIAL GEOGRAPHY AND SOCIO-SPATIAL INTERACTION

Agent-based models of social geography date to the mid-1970s, following popular work by Schelling (1971) and Sakoda (1971) with grid-based conceptual models of socio-spatial segregation dynamics. These models, while not originally computer-based and actually specified as CA, are significant in the social sciences as being exemplars of the utility of using simple rules among many interacting agents as a vehicle for exploring socio-spatial dynamics. Both Schelling and Sakoda developed very simple models of segregation between binary groups over space and evaluated the conditions under which populations could be polarized following the complex emergence of biases in their feelings toward groups of an opposite kind and the tipping-point (what is known as a phase shift in the complexity literature) at which socio-spatial polarization would occur. Indeed, Schelling shared the Nobel Prize in economics in 2005 for related work on the dynamics of conflict and cooperation. More recent work in this area by geographers has focused on modeling micro-scale segregation dynamics in residential choice (Torrens and Nara 2007b) and examination of the computational geometry of agents' neighborhood filters for satisfaction with their residential choices (Benenson et al. 2002).

4.4 ECONOMIC GEOGRAPHY

Agent-based models are popularly used in economic research (Farmer and Foley 2009) and spatial ABMs are used to infuse geographic context into economic or econometric simulations, for assessing the role of location, space, place, proximity, distance and scaling on economic processes. Often, modelers are interested in the influence of geography on the complexity of economic systems (Schweitzer 2002). Krugman (1996), for example, has developed CA models of international trade and urban agglomeration economies based on spatial ABMs. ABMs are also widely used in urban economics, where they allow for detailed representation of spatial decision-making and its relationship to community-level economic phenomena in residential property markets (Benenson and Torrens 2004; Torrens 2007b), gentrification (Torrens and Nara 2007b), land development (Semboloni et al. 2004), and even gasoline pricing (Heppenstall et al. 2005).

4.5 PHYSICAL GEOGRAPHY

Agent-based approaches are also employed in physical geography, where agents are designed to mimic physical components of geographical systems in, for example, geomorphology and hydrology. In these cases, agents often represent elementary particles in a physical system. Their 'agency' may be described in terms of the physical factors that determine their state, as in phase transition between ice, vapor and water. Rules may also be derived to explain how agents interact at small geographies to determine larger systems, such as the interactive behavior of snowflakes in blizzards (Kronholm and Birkeland 2005) or rock particles in sediment flow (D'Ambrosio et al. 2003; Vandewalle and Galam

1999). Use of agent-based modeling to explore the interactions of 'humans' as agents within physical environments and as agents of change – particularly regarding coupled human-natural systems (Balram and Dragićević 2003) and issues such as landscape erosion (Wainwright and Mulligan 2004), deforestation (Wainwright and Mulligan 2004), and land cover change (Veldkamp and Lambin 2001) – within those environments is growing in popularity.

4.6 HAZARDS

Agent-based models have also seen use for geographical research as a methodology for simulating risk and vulnerability. Emergency and catastrophic events, by their very nature, are all but impossible to explore and experiment with in the real world. Models are therefore an invaluable tool in evaluating plans and scenarios for vulnerability and resilience. Such models need to be realistic to be useful, however and ABM methodologies can play an important role in modeling the human agency in panic, evacuation, response, and so forth. Vicsek and Farkas have published some of the seminal ABM work in modeling escape panic among crowd members in egress scenarios, using physics to explore bottlenecks at exits and the dynamic behavior of crowds as excitable media (Farkas et al. 2002). Helbing et al. have modeled the success of varying strategies for escape from buildings in emergency situations, from the perspective of individual and collective human behavior (Helbing et al. 2000). Transportation modelers have also begun to use agent-like mechanisms as a component of decision and planning support systems, to better refine their representation of evacuation behavior, in wildfire events, for example (Cova and Johnson 2002).

4.7 SPATIAL EPIDEMIOLOGY

Agent-based tools have also recently been used to extend the reach of classic diffusion models in epidemiology research (Epstein 2009) to the scale of individual households (Barrett et al. 2005). Geographically-explicit ABMs, developed as Geographic Automata Systems (Torrens and Benenson 2005) in GIScience are now being used in the veterinary sciences to bolster the ability of models to trace likely paths for pathogens such as foot-and-mouth disease (Ward et al. 2007). Other researchers are also making use of spatial science in the development of ABMs for the biology of disease within the body. This is particularly true of systems biology and computational biology (Kitano 2002b, 2002a; Noble 2002), where developments in ABMs of local-scale biology are catalyzing new insights in the simulation of tumor growth, for example (Deutsch and Dormann 2004; Ermentrout and Edelstein-Keshet 1993).

4.8 BEHAVIORAL GEOGRAPHY

Agent-based modeld (ABMs) are also being used as models of spatial cognition for applications in behavioral geography (Frank et al. 1992; Golledge and Stimson 1997; Mark et al. 1999; Torrens 2007a). This trend is related, in part, to recent development of computational forms of semantic analysis in Geographic Information Systems and for Web services on the Geoweb (Torrens 2009; Yu and Peuquet 2009; Zhang and Tsou 2009). Uptake of ABMs is this area is particularly keen in investigation of formal models for spatial way-finding (Raubal 2001b; Torrens 2007a).

5 The Future of ABMs and Modeling in the Spatial Sciences

The use of ABMs in the spatial sciences, while growing in popularity, is still in a stage of relative infancy as a topic of academic inquiry. Some relatively serious challenges remain in advancing the research agenda further.

Much development work with ABMs in the spatial sciences is focused on the development of methods and tools. Relatively little work is performed with theory-building as a primary objective (Epstein 2006). This is understandable given the relative nascency of the methodology. There is a more long-standing problem, however, with the relationship between ABMs and theory. Much geographical theory abstracts from the micro-scale. However (and essentially put), we often lack the theory to derive rules for ABMs that treat individuals and scale all the way up to large macro-systems (Batty 2005).

Geographers encounter related problems with data and dataware to support ABMs. Detailed data-sets that might be used to parameterize and calibrate fine-scaled ABMs are rarely available, owing to privacy concerns of sharing data about individuals, the difficulties of collecting data longitudinally over long time periods and the qualitative training of generations of geographers used to thinking spatially but in aggregate terms. This is beginning to change, however; individual-scale data are routinely collected in transport studies for example, and geospatial technologies and cyberinfrastructure for automated tracking and sensor webs do provide near-real-time data feeds for whole study groups in some instances (Eagle and Pentland 2006; González et al. 2008; Lazer et al. 2009). Spatial scientists and other scientists are also turning to innovative methods for generating 'synthetic data populations' for use in ABMs (Bush 2001).

Validating ABMs is hugely problematic because quantitative geography has not focused on development of techniques for measuring and analyzing individual units as part of massively interactive, dynamic and non-linear systems (Batty and Torrens 2005; Pontius et al. 2007). This has led to some criticism of ABMs as 'toy models' in the absence of robust quantitative schemes that would allow for agent-based simulations to be registered to or compared with real-world systems (Couclelis 2002; Faith 1998). A related complication is that ABMs are often allied with complex adaptive systems, yet research into spatial complexity has gained momentum only in recent years (Manson 2007). This has led to difficulties in determining the signatures of complexity in ABMs and the real-world phenomena that they are used to simulate. In particular, it has been quite difficult for developers of ABMs to create a methodological structure for capturing novel spatial ensembles and facilitating adaptation and learning in their simulations. Many ABM developers in the spatial sciences have turned to GIScience in an attempt to overcome some of these difficulties, with some success thus far. The coupling of ABMs with Geographic Information Systems has provided a framework for structuring the design and application of ABMs for theory-experimentation, as well as providing a natural platform for integrating and reconciling diverse data-sets and validation schemes (Brown et al. 2005; Torrens and Benenson 2005).

6 Epilog

Agent-based models, although a recent introduction to the spatial sciences, have become increasingly popular in their use as a tool for model-building and applied simulation. Early work in developing ABMs for spatial science research was characterized by the adoption of tools from other fields, particularly from artificial life research

(Maes 1995) and related developments in physics and ecology (De Angelis and Gross 1992). Quite quickly, however, development of ABMs in the spatial sciences matured to the point that it began to have a reciprocal influence on these fields (Albrecht 2005). Many spatial ABMs are now used outside the spatial sciences, where the technique is often referred to as 'geosimulation' (Benenson and Torrens 2004): in computer science (Zhao and Murayama 2007), computer graphics and design (Ali and Moulin 2005), sociology (Koch 2003), machine-learning and data-mining (Filho et al. 2004), education and training (Furtado and Vasconcelos 2007), veterinary epidemiology (Ward et al. 2007), and criminology (Melo et al. 2006), for example. One significant aspect of this cross-fertilization through tool-building is the infusion of 'spatial thinking' from the spatial sciences to other disciplines (Albrecht 2005). Another is the potential for spatial ABMs to contribute to the emergence of next-generation Geographic Information Systems, built around semantic 'process models' (Torrens 2009; Egenhofer 2002).

These developments are in their relative infancy however and as mentioned earlier, in this article serious challenges remain to be overcome, particularly in allying ABMs to theory and data in ways that are extensible and meaningful.

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Short Biography

Dr. Paul M. Torrens is an Associate Professor in the School of Geographical Sciences and Urban Planning at Arizona State University and Director of its Geosimulation Laboratory. Paul is also an Affiliate in the University's Center for Social Dynamics and Complexity, as well as the GeoDa Center for Geospatial Analysis and Computation. His research is focused on Geographic Information Science and development of geosimulation and geocomputation tools, applied modeling of complex urban systems, and new emerging cyberspaces. His projects have been supported by the UK Economic and Social Research Council, the US National Science Foundation, Science Foundation Arizona, Autodesk, Inc., and Alias Research. His research has been published widely and his work has featured in a diverse array of outlets, from *Vanity Fair* and *Forbes* to *Popular Mechanics* and *Discover Magazine*. His work earned him a *CAREER* Award from the US National Science Foundation in 2007 and he was awarded the *Presidential Early Career Award for Scientists and Engineers* by President George W. Bush in 2008. The Presidential Early Career Award is the highest honor that the US government bestows upon young scientists; Torrens is the first geographer to receive the Award.

Note

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Appendix 1: A List of Agent-based Modeling Applications

Pedestrian Simulation

- STREETS model (Haklay et al. 2001)
- PEDFLOW model (Kukla et al. 2001)
- Notting Hill carnival model (Batty et al. 2003)
- Social force model (Helbing and Molnár 1995)
- Level of service models (Blue and Adler 2001; Gipps and Marksjö 1985)
- Space syntax models (Turner and Penn 2002)
- Recreation models (Gimblett 2005; Itami 1988, 2002; Itami et al. 2003)

Crowd Simulation

- Animated urban models (Musse and Thalmann 1997; Pelechano et al. 2005; Tecchia et al. 2002; Torrens 2007a; Treuille et al. 2006)
- Machine-learning models (Lee et al. 2007; Lerner et al. 2007, 2009)
- Procedural crowds (Haciomeroglu et al. 2008; Ken et al. 2008; Prescott et al. 2004; Ulicny et al. 2004; Yersin et al. 2009; Yeh et al. 2008)

Evacuation Models

- EXODUS (Galea et al. 1996; Gwynne et al. 1999)
- Legion (Berrou et al. 2007)
- HiDAC (High-Density Autonomous Crowds) (Pelechano and Badler 2006)
- Physics models (Gwynne et al. 1999; Vicsek 2003)

Sociology

- The Brookings Civil Violence Model (Epstein 2002)
- Sugarscape (Epstein and Axtell 1996)
- Schelling model (Schelling 1969)
- Sakoda model (Sakoda 1971)
- Residential segregation (Benenson et al. 2002)

Traffic Models

- TRANSIMS (USA) (Barrett et al. 1999; Nagel et al. 1998)
- TRANSIMS (Europe) (Cetin et al. 2002)
- MATSIM (Balmer et al. 2009)
- PARAMICS (Wylie et al. 1993)

Urbanization

- SLEUTH (Clarke et al. 1997, 2007)
- Sprawlsim (Torrens 2006, 2008)
- Dynamic Urban Evolution Model (DUEM) (Batty and Xie 1997; Batty et al. 1999, 2007; Xie 1996)
- MURBANDY (Engelen et al. 1995, 2002; White and Engelen 2000)
- The Cardiff model (Wu and Webster 1998, 2000)

- SIMPOP (Sanders et al. 1997)
- Urbanism (Waddell 2002)

Disease and Epidemiology

- EPISIMS (Barrett et al. 2005; Eubank et al. 2004)
- GeoGraph (Dibble and Feldman 2004)
- Tumor models (Deutsch and Dormann 2004; Malleta and De Pillis 2006)

Gentrification

- GraphCA (O'Sullivan 2002)
- Salt Lake City model (Torrens and Nara 2007a)

Artificial Life

- Boids (Reynolds 1987, 1993, 1999, 2006)
- Animats (Magnenat-Thalmann and Thalmann 1994; Meyer and Guillot 1994; Sims 1994; Terzopoulous et al. 1994)

Land-use and Land Cover Change

- SLUCE (Brown et al. 2002, 2005)
- FEARLUS (Polhill et al. 2001)
- Steven Manson (Manson 2000, 2005; Manson and Evans 2007)

Way-finding and Navigation

- Ontological models (Frank et al. 2001a; Raubal 2001a, b)
- Navigation meshes (Gayle et al. 2009; Lamarche and Donikian 2004; Nieuwenhuisen et al. 2007; Paris et al. 2007; Pettré et al. 2006; Salomon et al. 2003; Sud et al. 2008; van den Berg et al. 2008; Yersin et al. 2005)

Bots

- GeoAgents (Yu and Peuquet 2009)
- Web services (Zhang and Tsou 2009)

Economic Geography

- CityDev (Semboloni et al. 2004)
- Gasoline pricing (Heppenstall et al. 2005)
- Urban agglomeration (Batty 2001; Fujita et al. 2001; Krugman 1996)

Physical Geography

- Avalanches (Kronholm and Birkeland 2005)
- Sediment flow (D'Ambrosio, Di Gregorio and Iovine 2003; Vandewalle and Galam 1999)

- Human–environment interaction (Balram and Dragićević 2003)
- Landscape erosion (Wainwright and Mulligan 2004)

Appendix 2: Development Environments for ABMs

An overview (Railsback et al. 2006)

- RePast (North et al. 2006)
- NetLogo (Blikstein et al. 2005)
- Swarm (Luna and Stefansson 2000; Minar et al. 1996; Stefansson 1997, 2000)
- TRANSIMS (Barrett et al. 1999; Cetin et al. 2001; Nagel et al. 1998; Nagel and Rickert 2001)
- MATSIM (Balmer et al. 2009)
- DUEM (Batty and Xie 1997; Batty et al. 1999, 2007; Xie 1996)
- SLEUTH (Clarke et al. 1997, 2007)
- MAML (Gulyás et al. 1999)
- MURBANDY (Engelen et al. 2002)
- Urbansim (Waddell 2002)

References

Albrecht, J. (2005). A new age for geosimulation. *Transactions in Geographic Information Science* 9 (4), pp. 451–454.

Ali, W. and Moulin, B. (2005). 2D-3D multiagent geosimulation with knowledge-based agents of customers' shopping behavior in a shopping mall. In: Cohn, A. G. and Mark, D. M. (eds) *Lecture notes in computer science 3693: Proceedings of the Conference on Spatial Information Theory*. Heidelberg: Springer, pp. 445–458.

Alonso, W. (1960). A theory of the urban land market. *Papers and Proceedings of the Regional Science Association* 6, pp. 149–158.

Anderies, J. M., Janssen, M. A. and Ostrom, E. (2004). A framework to analyze the robustness of social-ecological systems from an institutional perspective. *Ecology and Society* 9 (1), p. 18.

Arthur, W. B. (1994). Inductive reasoning and bounded rationality. *The American Economic Review* 84 (2), pp. 406–411.

Balmer, M., et al. (2009). MATSim-T: architecture and simulation times. In: Bazzan, A. L. C. and Klügl, F (eds) *Multi-agent systems for traffic and transportation*. Hershey, PA: IGI Global.

Balram, S. and Dragićević, S. (2003). Integrating geographic information systems and agent-based modeling techniques for simulating social and ecological processes. *The Professional Geographer* 55 (2), pp. 301–302.

Balzter, H., Braun, P. W. and Koehler, W. (1998). Cellular automata models for vegetation dynamics. *Ecological Modelling* 107, pp. 113–125.

Barrett, C. L., et al. (1999). *TRANSIMS (TRansportation ANalysis SIMulation System)*. Volume 0: Overview. Los Alamos National Laboratory Report LA-UR-99-1658. Los Alamos: Los Alamos National Laboratory.

Barrett, C. L., Eubank, S. G. and Smith, J. P. (2005). If smallpox strikes Portland. *Scientific American* 292 (3), pp. 42–49.

Batty, M. (1997). Cellular automata and urban form: a primer. *Journal of the American Planning Association* 63 (2), pp. 266–274.

Batty, M. (2001). Polynucleated urban landscapes. *Urban Studies* 38 (4), pp. 635–655.

Batty, M. (2005). *Cities and complexity: understanding cities with cellular automata, agent-Based models, and fractals*. Cambridge, MA: The MIT Press.

Batty, M. and Torrens, P. M. (2005). Modeling and prediction in a complex world. *Futures* 37 (7), pp. 745–766.

Batty, M. and Xie, Y. (1997). Possible urban automata. *Environment and Planning B* 24, pp. 175–192.

Batty, M., Xie, Y. and Sun, Z. (1999). Modeling urban dynamics through GIS-based cellular automata. *Computers, Environment and Urban Systems* 23 (3), pp. 205–233.

Batty, M., Desyllas, J. and Duxbury, E. (2003). The discrete dynamics of small-scale spatial events: Agent-based models of mobility in carnivals and street parades. *International Journal of Geographical Information Science* 17 (7), pp. 673–697.

Batty, M., Xie, Y. and Zhao, K. (2007). Simulating emergent urban form using agent-based modeling: Desakota in the Suzhou-Wuxian region in China. *Annals of the Association of American Geographers* 97 (3), pp. 477–495.

Benenson, I. (1999). Modelling population dynamics in the city: from a regional to a multi-agent approach. *Discrete Dynamics in Nature and Society* 3, 149–170.

Benenson, I. and Torrens, P. M. (2004). *Geosimulation: Automata-based modeling of urban phenomena*. London: John Wiley & Sons.

Benenson, I., Omer, I. and Hatna, E. (2002). Entity-based modeling of urban residential dynamics: the case of Yaffo, Tel Aviv. *Environment and Planning B: Planning and Design* 29, pp. 491–512.

van den Berg, J., et al. (2008). Interactive navigation of multiple agents in crowded environments. *Proceedings of Proceedings of the 2008 symposium on Interactive 3D graphics and games*, Redwood City, CA, pp. 139–147.

Berger, T. (2001). Agent-based spatial models applied to agriculture: a simulation tool for technology diffusion, resource use changes, and policy analysis. *Agricultural Economics* 25, pp. 245–260.

Berrou, J., et al. (2007). Calibration and validation of the Legion simulation model using empirical data. In: *Pedestrian and Evacuation Dynamics 2005*. Berlin: Springer, pp. 167–181.

Blikstein, P., Abrahamson, D. and Wilensky, U. (2005). NetLogo: where we are, where we're going. *Proceedings of Proceedings of the Annual Meeting of Interaction Design & Children*, Boulder, CO, June 8–10, 2005, Eisenberg, M. and Eisenberg, A. (eds).

Blue, V. and Adler, J. (2001). Cellular automata microsimulation for modeling bi-directional pedestrian walkways. *Transportation Research Part B* 35, pp. 293–312.

Bonabeau, E. (2002). Agent-based modeling: methods and techniques for simulating human systems. *Proceedings of the National Academy of Sciences* 99 (3), pp. 7280–7287.

Bonabeau, E., Dorigo, M. and Theraulaz, G. (1999). *Swarm intelligence: from natural to artificial systems, Santa Fe Institue studies in the sciences of complexity*. New York: Oxford University Press.

Brown, D. G., Page, S. E., Riolo, R. and Rand, W. (2002). Modeling the effects of greenbelts on the urban-rural fringe. *Proceedings of iEMSs: Integrated Assessment and Decision Support*, Lugano, Switzerland, June 24–27, 2002.

Brown, D. G., et al. (2005). Spatial process and data models: Toward integration of agent-based models and GIS. *Journal of Geographical Systems* 7 (1), pp. 25–47.

Bush, B. W. (2001). *Portland Synthetic Population*. Los Alamos National Laboratory Report LA-UR-00-5972. Los Alamos: Los Alamos National Laboratory. January 7–11, 2001.

Cetin, N., Nagel, K., Raney, B. and Voellmy, A. (2001). Large-scale multi-agent transportation systems. *Computational Physics Communications* 147 (1–2), pp. 559–564.

Cetin, N., Nagel, K., Raney, B. and Voellmy, A. (2002). Large-scale multi-agent transportation simulations. *Computer Physics Communications* 147 (1–2), pp. 559–564.

Chapin, F. S. and Weiss, S. F. (1962). *Factors influencing land development. An Urban Studies Research Monograph*. Chapel Hill: Center for Urban and Regional Studies, Institute for Research in Social Science, University of North Carolina.

Clarke, K. C., Hoppen, S. and L. Gaydos, L. (1997). A self-modifying cellular automaton model of historical urbanization in the San Francisco Bay area. *Environment and Planning B* 24, pp. 247–261.

Clarke, K. C., Gazulis, N., Dietzel, C. and Goldstein, N. C. (2007). A decade of SLEUTHing: lessons learned from applications of a cellular automaton land use change model. In: Fisher, P. (ed.) *Classics in IJGIS: Twenty years of the International Journal of Geographical Information Science and Systems*. Boca Raton, FL: CRC Press, pp. 413–427.

Couclelis, H. (2002). Why I no longer work with agents: A challenge for ABMs of human-environment interactions. In: Parker, D., Berger, T. and Manson, S. M. (eds) *Agent-based models of land use and land cover change*. Bloomington, IN: LUCC, pp. 3–5.

Cova, T. J. and Johnson, J. P. (2002). Microsimulation of neighborhood evacuations in the urban-wildland interface. *Environment and Planning A* 34, pp. 2211–2229.

D'Ambrosio, D., Di Gregorio, S. and Iovine, G. (2003). Simulating debris flows through a hexagonal Cellular Automata model: Sciddica S3-hex. *Natural Hazards and Earth System Sciences* 3, pp. 545–559.

De Angelis, D. and Gross, L. J. (1992). *Individual-based models and approaches in ecology: populations, communities, and ecosystems*. New York: Chapman and Hall.

Deutsch, A. and Dormann, S. (2004). *Cellular automaton modeling of biological pattern formation*. Boston: Birkhauser.

Dibble, C. and Feldman, P. G. (2004). The GeoGraph 3D computational laboratory: network and terrain landscapes for RePast. *Journal of Artificial Societies and Social Simulation* 7 (1). [Online]. Retrieved on 01 Jan 2009 from http://jasss.soc.surrey.ac.uk/7/1/7.html.

DiBiase, D., et al. (2006). *GIS&T body of knowledge*. Washington D.C.: Association of American Geographers.

Dijkstra, E. W. (1959). A note on two problems in connection with graphs. *Numerische Mathematik* 1, pp. 269–271.

Eagle, N. and Pentland, A. (2006). Reality mining: sensing complex social systems. *Personal and Ubiquitous Computing* 10 (4), pp. 255–268.

Egenhofer, M. J. (2002). Toward the semantic geospatial web. In: *Proceedings of the 10th ACM international symposium on Advances in geographic information systems*. McLean, Virginia, USA: ACM.

Engelen, G., White, R., Uljee, I. and Drazan, P. (1995). Using cellular automata for integrated modelling of socio-environmental systems. *Environmental Monitoring and Assessment* 30, pp. 203–214.

Engelen, G., White, R. and Uljee, I. (2002). *The MURBANDY and MOLAND models for Dublin*. Maastricht: Research Institute for Knowledge Systems (RIKS) BV.

Epstein, J. M. (2002). Modeling civil violence: an agent-based computational approach. *Proceedings of the National Academy of Science* 99 (3), pp. 7243–7250.

Epstein, J. M. (2006). *Generative social science: studies in agent-based computational modeling, Princeton studies in complexity*. Princeton, NJ: Princeton University Press.

Epstein, J. M. (2009). Modelling to contain pandemics. *Nature* 460 (7256), p. 687.

Epstein, J. M. and Axtell, R. (1996). *Growing artificial societies from the bottom up*. Washington D.C.: Brookings Institution.

Ermentrout, G. B. and Edelstein-Keshet, L. (1993). Cellular automata approaches to biological modeling. *Journal of Theoretical Bilogy* 160, pp. 97–133.

Eubank, S., et al. (2004). Modelling disease outbreaks in realistic urban social networks. *Nature* 429 (6988), pp. 180–184.

Faith, J. (1998). Why gliders don't exist: anti-reductionism and emergence. In: Adami, C. (ed.) *Artificial Life VI: Proceedings of the Sixth International Conference on Artificial Life*. Cambridge, MA: MIT Press.

Farkas, I., Helbing, D. and Vicsek, T. (2002). Crowd behaves as excitable media during Mexican wave. *Nature* 419, p. 131.

Farmer, J. D. and Foley, D. (2009). The economy needs agent-based modeling. *Nature* 460 (7256), pp. 685–686.

Fayyad, U., Piatetsky-Shapiro, G. and Smyth, P. (1996). The KDD process for extracting useful knowledge from volumes of data. *Communications of the ACM* 39 (11), pp. 27–34.

Ferber, J. (1999). *Multi-agent systems: an introduction to distributed artificial intelligence*. Harlow (UK): Addison-Wesley.

Filho, E. V., Pinheiro, V. and Furtado, V. (2004). Mining data and providing explanation to improve learning in geosimulation. In: Vicari, R. M. and Paraguaçu, F. (eds) *Lecture notes in computer science 3220: Intelligent Tutoring Systems*. Berlin: Springer, pp. 821–823.

Fonte, C. C. and Lodwick, W. A. (2004). Areas of fuzzy geographical entities. *International Journal of Geographic Information Science* 18 (2), pp. 127–150.

Fotheringham, A. S. and O'Kelly, M. E. (1989). *Spatial interaction models: formulations and applications, studies in operational regional science*. Dordrecht: Kluwer Academic Publishers.

Frank, A. A. U., Campari, I. and Formentini, U. (eds) (1992). *Theories and methods of spatio-temporal reasoning in geographic space*. Berlin: Springer-Verlag.

Frank, A., Bittner, S. and Raubal, M. (2001a). Spatial and cognitive simulation with Multi-Agent Systems. *Proceedings of Spatial Information Theory: Foundations of Geographic Information Science International Conference, COSIT 2001*, Morro Bay, CA, September 19–23, Goos, G., Hartmanis J. and van Leeuwen, J. (eds).

Frank, A., Bittner, S. and Raubal, M. (2001b). Spatial and cognitive simulation with multi-agent systems. In: Montello, D. R. (ed.) *Spatial information theory*. Berlin: Springer-Verlag, pp. 124–139.

Fujita, M., Krugman, P. and Venables, A. J. (2001). *The spatial economy: cities, regions, and international trade*. Cambridge, MA: The MIT Press.

Furtado, V. and Vasconcelos, E. (2007). Geosimulation in education: a system for teaching police resource allocation. *International Journal of Artificial Intelligence in Education* 17 (1), pp. 57–81.

Galea, E. R., Owen, M. and Lawrence, P. (1996). The EXODUS Model. *Fire Engineers Journal* (July) 56(183), pp. 26–30.

Gayle, R., et al. (2009). Interactive navigation of heterogeneous agents using adaptive roadmaps. *IEEE Transactions on Visualization and Computer Graphics* 15 (1), pp. 34–48.

Gimblett, R. (2005). Modelling human-landscape interactions in spatially complex settings: Where are we and where are we going? In: Zerger, A. and R. M. Argent, R. M. (eds) *MODSIM: International Congress on Modelling and Simulation*. Melbourne: The Modelling and Simulation Society of Australia and New Zealand.

Gipps, P. G. and Marksjö, B. (1985). A microsimulation model for pedestrian flows. *Mathemathics and Computers in Simulation* 27, pp. 95–105.

Golledge, R. and Stimson, R. J. (1997). *Spatial behavior: a geographic perspective*. New York: The Guilford Press.

González, M. C., Hidalgo, C. A. and Barabási, A.-L. (2008). Understanding individual human mobility patterns. *Nature* 453 (7196), pp. 779–782.

Griffin, W. A. (2005). Using agent based modeling to simulate the influence of family level stress on disease progression. In: Crane, D. R. and Marshall, E. S. (eds) *Handbook of Families and Health: Interdisciplinary Perspectives*. Thousand Oaks, CA: Sage Publications, pp. 291–315.

Grimm, V. (1999). Ten years of individual-based modelling in ecology: what have we learned and what could we learn in the future. *Ecological Modelling* 115, pp. 129–148.

Gulyás, L., Kozsik, T. and Corliss, J. B. (1999). The Multi-Agent Modelling Language and the Model Design Interface. *Journal of Artificial Societies and Social Simulation* 2(3). [Online]. Retrieved on 01 Jan 2009 from http://jasss.soc.surrey.ac.uk/2/3/8.html.

Gurney, K. (1997). *An introduction to neural networks*. London: Routledge.

Gwynne, S., et al. (1999). A review of the methodologies used in the computer simulation of evacuation from the built environment. *Building and Environment* 34, pp. 741–749.

Haciomeroglu, M., Laycock, R. G. and Day, A. M. (2008). Dynamically populating large urban environments with ambient virtual humans. *Computer Animation and Virtual Worlds* 19 (3–4), pp. 307–317.

Hägerstrand, T. (1967). *Innovation diffusion as a spatial process*. Translated by A. Pred. Chicago: University of Chicago Press.

Haklay, M., O'Sullivan, D., Thurstain-Goodwin, M. and Schelhorn, T. (2001). ''So go downtown'': simulating pedestrian movement in town centres. *Environment and Planning B* 28 (3), pp. 343–359.

Helbing, D. and Molnár, P. (1995). Social force model for pedestrian dynamics. *Physical Review E* 51, pp. 4282–4286.

Helbing, D., Illés, F., et al. (2000). Simulating dynamical features of escape panic. *Nature* 407(September 28): 487–490.

Henderson, L. F. (1971). The statistics of crowd fluids. *Nature* 229 (5284), pp. 381–383.

Hensher, D. A., Button, K. J., Haynes, K. E. and Stopher, P. (2004). *Handbook of transport geography and spatial system, Vol. 5, Handbooks in transport* London: Elsevier.

Heppenstall, A. J., Evans, A. J. and Birkin, M. H. (2005). A hybrid multi-agent/spatial interaction model system for petrol price setting. *Transactions in GIS* 9 (1), pp. 35–51.

Hughes, R. L. (2003). The flow of human crowds. *Annual Review of Fluid Mechanics* 35, pp. 169–182.

Itami, R. (1988). Cellular worlds: models for dynamic conception of landscapes. *Landscape Architecture* 78 (5), pp. 52–57.

Itami, R. (2002). Mobile agents with spatial intelligence. In: Gimblett, H. R. (ed.) *Integrating Geographic Information Systems and agent-based modeling for simulating social and ecological processes*. Oxford: Oxford University Press, pp. 191–210.

Itami, R., et al. (2003). RBSim 2: Simulating the complex interactions between human movement and the outdoor recreation environment. *Journal for Nature Conservation* 11, pp. 279–296.

Ken, P., Chris, H., Craig, R. and Friedrich, K. (2008). Four views of procedural character animation for computer games. In: *Proceedings of the 2008 ACM SIGGRAPH symposium on Video games*. Los Angeles, California: ACM.

Kitano, H. (2002a). Computational systems biology. *Nature* 420 (6912), pp. 206–210.

Kitano, H. (2002b). Systems biology: a brief overview. *Science* 295 (5560), pp. 1662–1664.

Klir, G. J. and Yuan, B. (1994). *Fuzzy sets and fuzzy logic: theory and applications*. Upper Saddle River, NJ: Prentice-Hall.

Koch, A. (2003). Klagenfurter Geographische Schriften 23: Sozialgeographische agentenbasierte geosimulation: Zur komplementarität von raumsemantik und raummodell. In: Koch, A. and Mandl, P. (eds) *Multi-Agenten-Systeme in der Geographie*. Klagenfurt: Institut für Geographie und Regionalforschung der Universität Klagenfurt, pp. 35–64.

Kohler, T. A. and Gumerman, G. (2001). *Dynamics in human and primate cocieties*. New York: Oxford University Press.

Kronholm, K. and Birkeland, K. W. (2005). Integrating spatial patterns into a snow avalanche cellular automata model. *Geophysical. Research Letters* 32 (19), pp. L19504.1–L19504.4.

Krugman, P. (1996). *The self-organizing economy*. Malden, MA: Blackwell.

Kukla, R., Kerridge, J., Willis, A. and Hine, J. (2001). PEDFLOW: development of an autonomous agent model of pedestrian flow. *Transportation Research Record: Journal of the Transportation Research Board* 1774, pp. 11–17.

Lamarche, F. and Donikian, S. (2004). Crowd of virtual humans: a new approach for real time navigation in complex and structured environments. *Computer Graphics Forum* 23 (3), pp. 509–518.

Lazer, D., et al. (2009). Computational social science. *Science* 323 (5915), pp. 721–723.

Lee, K. H., Choi, M. G., Hong, Q. and Lee, J. (2007). Group behavior from video: a data-driven approach to crowd simulation. In: *Proceedings of the 2007 ACM SIGGRAPH/Eurographics symposium on Computer animation*. San Diego, California: Eurographics Association.

Leonard, A. (1997). *Bots: the origin of a new species*. San Francisco: Hardwired.

Lerner, A., Chrysanthou, Y. and Lischinski, D. (2007). Crowds by Example. *Computer Graphics Forum* 26, pp. 655–664.

Lerner, A., Fitusi, E., Chrysanthou, Y. and Cohen-Or, D. (2009). Fitting behaviors to pedestrian simulations. *Proceedings of Eurographics/ ACM SIGGRAPH Symposium on Computer Animation*, New Orleans, LA, August 1–2, 2009, Grinspun, E. and Hodgins, J. (eds).

Li, X. and Yeh, A. G.-O. (2002). Neural-network-based cellular automata for simulating multiple land use changes using GIS. *International Journal of Geographical Information Science* 16 (4), pp. 323–343.

Luna, F. and Stefansson, B. (eds). (2000). *Economic simulation in swarm: agent-based modelling and object oriented programming*. Dordrecht: Kluwer.

Macy, M. W. and Willer, R. (2002). From factors to actors: computational sociology and agent-based modeling. *Annual Review of Sociology* 28, pp. 143–166.

Maes, P. (1995). Artificial Life meets entertainment: life like autonomous agents. *Communications of the ACM* 38 (11), pp. 108–114.

Magnenat-Thalmann, N. and Thalmann, D. (1994). Introduction: creating artificial life in virtual reality. In: Magnenat-Thalmann, N. and Thalmann, D. (eds) *Artificial life and virtual reality*. New York: John Wiley and Sons.

Malleta, D. G. and De Pillis, L. G. (2006). A cellular automata model of tumor-immune system interactions. *Journal of Theoretical Biology* 239 (3), pp. 334–350.

Manson, S. M. (2000). Agent-based dynamic spatial simulation of land-use/land cover change in the Yucatán Peninsula, Mexico. *Proceedings of Fourth International Conference on Integrating GIS and Environmental Modeling (GIS/EM4): Problems, Prospects and Research Needs*, Banff, Alberta, Canada, September 2–8, 2000.

Manson, S. M. (2001). Simplifying complexity: a review of complexity theory. *Geoforum* 32 (3), pp. 405–414.

Manson, S. M. (2005). Agent-based modeling and genetic programming for modeling land change in the Southern Yucatan Peninsular Region of Mexico. *Agriculture, Ecosystems and Environment* 111 (1), pp. 47–62.

Manson, S. M. (2007). Challenges in evaluating models of geographic complexity. *Environment and Planning B* 34 (2), pp. 245–260.

Manson, S. M. and Evans, T. (2007). Agent-based modeling of deforestation in southern Yucatán, Mexico, and reforestation in the Midwest United States. *Proceedings of the National Academy of Sciences* 104 (52), pp. 20678–20683.

Mark, D. M., et al. (1999). Cognitive models of geographic space. *International Journal of Geographic Information Science* 13 (8), pp. 747–774.

McFadden, D. (1974). Conditional logit analysis of qualitative choice behavior. In: Zarembka, P. (ed.) *Frontiers in Econometrics*. New York: Academic Press, pp. 105–142.

Melo, A., Menezes, R., Furtado, V. and Coelho, A. L. V. (2006). Self-organized and social models of criminal activity in urban environments. In: Dorigo, M., Gambardella, L. M., Birattari, M., Martinoli, A., Poli, R. and Stützle, T. (eds) *Lecture Notes in Computer Science 4150: Ant Colony Optimization and Swarm Intelligence*. Berlin: Springer, pp. 518–519.

Meyer, J.-A. and Guillot, A. (1994). From SAB90 to SAB94: four years of animat research. In: Cliff, D., Husbands, P., Meyer, J.-A. and Wilson, S. (eds). *From Animals to animats 3. Proceedings of the Third International Conference on Simulation of Adaptive Behavior*. Cambridge, MA: The MIT Press, pp. 2–11.

Minar, N., Burkhart, R., Langton, C. and Askenazi, M. (1996). *The Swarm simulation system: a toolkit for building multi-agent simulations*. Working paper. Santa Fe: Santa Fe Institute.

Mitchell, M. (1998). *An introduction to genetic algorithms, complex adaptive systems*. Cambridge, MA: MIT Press.

Musse, S. R. and Thalmann, D. (1997). A model of human crowd behavior: group inter-relationship and collision detection analysis. In: *Computer Animation and Simulations '97, Proceedings of the Eurographics Workshop*. Budapest: Springer-Verlag, pp. 39–51.

Nagel, K. and Rickert, M. (2001). Parallel implementation of the TRANSIMS micro-simulation. *Parallel Computing* 27 (12), pp. 1611–1639.

Nagel, K. and Schreckenberg, M. (1992). A cellular automaton model for freeway traffic. *Journal de Physique I* 2 (12), pp. 2221–2229.

Nagel, K., et al. (1998). Regional transportation simulations. In: Tenter, A. (ed.) *High Performance Computing*. San Diego: The Society for Computer Simulation International, pp. 104–109.

Nakajima, T. (1977). Application de la théorie de l'automate à la simulation de l'évolution de l'espace urbain. In *Congrès Sur La Méthodologie De L'Aménagement Et Du Dévelopment*. Montreal: Association Canadienne-Française Pour L'Avancement Des Sciences et Comité De Coordination Des Centres De Recherches En Aménagement, Développement Et Planification (CRADEP), pp. 154–160.

von Neumann, J. (1951). The general and logical theory of automata. In: Jeffress, L. A. (ed.) *Cerebral mechanisms in behavior*. New York: Wiley, pp. 1–41.

Newton, I. (1687). *Philosophiae Naturalis Principia Mathematica*. London: Royal Society of London for the Improvement of Natural Knowledge .

Nieuwenhuisen, D., Kamphuis, A. and Overmars, M. H. (2007). High quality navigation in computer games. *Sci. Comput. Program*. 67 (1), pp. 91–104.

Noble, D. (2002). Opinion: the rise of computational biology. *Nature Reviews Molecular Cell Biology* 3 (6), pp. 459–463.

North, M. J., Collier, N. T. and Vos, J. R. (2006). Experiences creating three implementations of the repast agent modeling toolkit. *ACM Trans. Model. Comput. Simul*. 16 (1), pp. 1–25.

O'Sullivan, D. (2002). Toward micro-scale spatial modeling of gentrification. *Journal of Geographical Systems* 4 (3), pp. 251–274.

Paris, S., Pettré, J. and Donikian, S. (2007). Pedestrian reactive navigation for crowd simulation: a predictive approach. *Computer Graphics Forum* 26 (3), pp. 665–674.

Pelechano, N. and Badler, N. I. (2006). Modeling crowd and trained leader behavior during building evacuation. *IEEE Computer Graphics and Applications* 26 (6), *pp*. 80–86.

Pelechano, N., O'Brien, K., Silverman, B. and Badler, N. I. (2005). Crowd simulation incorporating agent psychological models, roles and communication. *Proceedings of First International Workshop on Crowd Simulation*, Lausanne, Switzerland November 24–25, 2005.

Pettré, J., et al. (2006). Real-time navigating crowds: scalable simulation and rendering. *Computer Animation and Virtual Worlds* 17 (3–4), pp. 445–455.

Polhill, J. G., Gotts, N. M. and Law, A. N. R. (2001). Imitative and non-imitative strategies in a land use simulation. *Cybernetics and Systems* 32, pp. 285–307.

Pontius Jr., R. G., et al. (2007). Accuracy assessment for a simulation model of Amazonian deforestation. *Annals of the Association of American Geographers* 97 (4), pp. 677–695.

Prescott, D., et al. (2004). Building crowds of unique characters. In: *ACM SIGGRAPH 2004 Sketches*. Los Angeles, California: ACM.

Railsback, S. F., Lytinen, S. L. and Jackson, S. K. (2006). Agent-based simulation platforms: review and development recommendations. *Simulation* 82 (9), pp. 609–623.

Raubal, M. (2001). Human wayfinding in unfamiliar buildings: a simulation with a cognizing agent. *Cognitive Processing* 2(3), pp. 363–388. .

Raubal, M. (2001b). Ontology and epistemology for agent-based wayfinding simulation. *International Journal of Geographic Information Science* 15 (7), pp. 653–665.

Reynolds, C. W. (1987). Flocks, herds, and schools: a distributed behavioral model. *Computer Graphics* 21 (4), pp. 25–34.

Reynolds, C. W. (1993). An evolved, vision-based behavioral model of coordinated group motion. In: Meyer, J.-A., Roitblat, H. L. and Wilson, S. W. (eds) *From animals to animats 2*. Cambridge, MA: The MIT Press, pp. 384–392.

Reynolds, C. W. (1999). Steering behaviors for autonomous characters. *Proceedings of Game Developers Conference*, San Jose, CA.

Reynolds, C. W. (2006). Big fast crowds on PS3. In: *Proceedings of the 2006 ACM SIGGRAPH symposium on Videogames*. Boston, Massachusetts: ACM.

Russell, S. and Norvig, P. (1995). *Artificial intelligence: a modern approach*. Upper Saddle River, New Jersey: Prentice-Hall.

Sakoda, J. M. (1971). The checkerboard model of social interaction. *Journal of Mathematical Sociology* 1, pp. 119–132.

Salomon, B., Garber, M., Lin, M. C. and Manocha, D. (2003). Interactive navigation in complex environments using path planning. In: *Proceedings of the 2003 Symposium on Interactive 3D Graphics, SI3D 2003*, 28–30 April 2003, Monterey, California. Washington, D.C.: Association of Computing Machinery, pp. 41–50.

Sanders, L., et al. (1997). SIMPOP: A multiagent system for the study of urbanism. *Environment and Planning B* 24, pp. 287–305.

Schelling, T. C. (1969). Models of segregation. *American Economic Review* 59 (2), pp. 488–493.

Schelling, T. C. (1971). Dynamic models of segregation. *Journal of Mathematical Sociology* 1, pp. 143–186.

Schelling, T. C. (1978). *Micromotives and nacrobehavior*. New York: WW Norton and Company.

Schelling, T. C. (2003). *Brownian agents and active particles*. Berlin: Springer-Verlag.

Schweitzer, F. (2002). Modeling migration and economic agglomeration with active Brownian particles. In: Schweitzer, F. (ed.) *Modeling complexity in economic and social systems*, Singapore: World Scientific Publishing Company, pp. 137–161.

Semboloni, F., et al. (2004). CityDev, an interactive multi-agents urban model on the web. *Computers, Environment and Urban Systems* 28 (1/2), pp. 45–65.

Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal* 27, 379–423 and 623–656.

Shao, W. and Terzopoulos, D. (2007). Autonomous pedestrians. *Graphical Models* 69, pp. 246–274.

Sims, K. (1994). Evolving virtual creatures. *Proceedings of Proceedings of the 21st Annual Conference on Computer Graphics and Interactive Techniques*, pp. 15–22

Stefansson, B. (1997). Swarm: an object-oriented simulation platform applied to markets and organizations. In: Angeline, P., Reynolds, R., McDonnell, J. and Eberhart, R. (eds) *Evolutionary programming VI*. New York: Springer-Verlag.

Stefansson, B. (2000). Simulating economic agents in Swarm: a short tutorial for economists and other social scientists. In: Luna, F. and Stefansson, B. (eds) *Economic simulation in swarm: agent-based modelling and object-Oriented Programming*. Boston: Kluwer Academic Publishers, pp. 3–61.

Sud, A., et al. (2008). Real-time path planning in dynamic virtual environments using multiagent navigation graphs. *IEEE Transactions on Visualization and Computer Graphics* 14 (3), pp. 526–538.

Tecchia, F., Loscos, C. and Chrysanthou, Y. (2002). Visualizing crowds in real-time. *Computer Graphics Forum* 21 (4), pp. 753–765.

Terzopoulous, D., Tu, X. and Grzeszczuk, R. (1994). Artificial fishes: autonomous location, perception, behavior, and learning in a simulated physical world. *Artificial Life* 1 (4), pp. 327–351.

Tesfatsion, L. (1997). How economists can get alife. In: Arthur, W. B., Durlaf, S. and Lane, D. (eds) *The economy as an evolving complex system II*. Reading, MA: Addison-Wesley, pp. 533–564.
Tobler, W. (1970). A computer movie simulating urban growth in the Detroit region. *Economic Geography* 46(2), pp. 234–240.
Torrens, P. M. (2005). Geosimulation approaches to traffic modeling. In: Stopher, P., Button, K., Haynes, K. and Hensher, D. (eds) *Transport Geography and Spatial Systems*. London: Pergamon, pp. 549–565.
Torrens, P. M. (2006). Simulating sprawl. *Annals of the Association of American Geographers* 96(2), pp. 248–275.
Torrens, P. M. (2007a). Behavioral intelligence for geospatial agents in urban environments. *Proceedings of IEEE Intelligent Agent Technology*, Fremont, CA, pp. 63–66.
Torrens, P. M. (2007b). A geographic automata model of residential mobility. *Environment and Planning B* 33 (2), pp. 200–222.
Torrens, P. M. (2008). A toolkit for measuring sprawl. *Applied Spatial Analysis and Policy* 1(1), pp. 5–36.
Torrens, P. M. (2009). Process models and next-generation geographic information technology. In: ESRI (ed.) *GIS Best Practices: Essays on Geography and GIS*. Redlands, CA: ESRI Press, pp 63–75.
Torrens, P. M. and Benenson, I. (2005). Geographic automata systems. *International Journal of Geographical Information Science* 19 (4), pp. 385–412.
Torrens, P. M. and Nara, A. (2007a). Modeling gentrification dynamics: a hybrid appraoch. *Computers, Environment and Urban Systems* 31(3), pp. 337–361.
Torrens, P. M. and Nara, A. (2007b). Modeling gentrification dynamics: a hybrid approach. *Computers, Environment and Urban Systems* 31(3), pp. 337–361.
Treuille, A., Cooper, S. and Popović, Z. (2006). Continuum crowds. *ACM Transactions on Graphics* 25(3), pp. 1160–1168.
Turing, A. M. (1936). On computable numbers, with an application to the *Entscheidungsproblem*. *Proceedings of the London Mathematical Society Series* 2(42), pp. 230–265.
Turing, A. M. (1938). Correction to: on computable numbers, with an application to the *Entscheidungsproblem*. *Proceedings of the London Mathematical Society Series* 2(43), pp. 544–546.
Turing, A. M. (1950). Computing machinery and intelligence. *Mind* 49, pp. 433–460.
Turner, A. and Penn, A. (2002). Encoding natural movement as an agent-based system: an investigation into human pedestrian behaviour in the built environment. *Environment and Planning B: Planning and Design* 29(4), pp. 473–490.
Ulam, S. (1969). *A collection of mathematical problems*. New York: Interscience.
Ulicny, B., de Heras, P. and Thalmann, D. (2004). Crowdbrush: interactive authoring of real-time crowd scenes. *Proceedings of ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, Grenoble, August 27–29, 2004.
Vandewalle, N. and Galam, S. (1999). Ripples versus giant dunes in a saltation-avalanche model. *International Journal of Modern Physics C* 10 (6), pp. 1071–1076.
Veldkamp, A. and Lambin, E. F. (2001). Predicting land-use change. *Agriculture, Ecosystems and Environment* 85, pp. 1–6.
Vicsek, T. (2003). Crowd control. *Europhysics News* 34 (2), pp. 45–49.
Waddell, P. A. (2000). A behavioural simulation model for metropolitan policy analysis and planning: residential location and housing market components of UrbanSim. *Environment and Planning B* 27(2), pp. 247–263.
Waddell, P. A. (2002). UrbanSim: modeling urban development for land use, transportation and environmental planning. *Journal of the American Planning Association* 68(3), pp. 297–314.
Wagner, D. F. (1997). Cellular automata and geographic information systems. *Environment and Planning B* 24, pp. 219–234.
Wainwright, J. and Mulligan, M. (eds). (2004). *Environmental modelling: finding simplicity in complexity*. Chichester: John Wiley & Sons.
Ward, M. P., Laffan, S. W. and Highfield, L. D. (2007). The potential role of wild and feral animals as reservoirs of foot-and-mouth disease. *Preventive Veterinary Medicine* 80(1), pp. 9–23.
Weiner, N. (1948). *Cybernetics*. Cambridge, MA: MIT Press.
White, R. and Engelen, G. (2000). High-resolution integrated modelling of the spatial dynamics of urban and regional systems. *Computers, Environment and Urban Systems* 24, pp. 383–400.
Wu, F. (1998). An experiment on the generic polycentricity of urban growth in a cellular automatic city. *Environment and Planning B* 25, pp. 731–752.
Wu, F. and Webster, C. J. (1998). Simulation of land development through the integration of cellular automata and multicriteria evaluation. *Environment and Planning B* 25, pp. 103–126.
Wu, F. and Webster, C. J. (2000). Simulating artificial cities in a GIS environment: urban growth under alternative regulation regimes. *International Journal of Geographical Information Science* 14(7), 625–648.
Wylie, B., Cameron, G., Matthew, W. and McArthur, D. (1993). PARAMICS: Parallel Microscopic Traffic Simulator. *Proceedings of Second European Connection Machine Users Meeting*, Meudon, Paris, 11–14 October.
Xie, Y. (1996). A generalized model for cellular urban dynamics. *Geographical Analysis* 28 (4), pp. 350–373.

Yeh, H., et al. (2008). Composite agents. *Proceedings of ACM SIGGRAPH / Eurographics Symposium on Computer Animation*, Dublin, Ireland, July 7–9, 2008.

Yersin, B., et al. (2005). Steering a virtual crowd based on a semantically augmented navigation graph. *Proceedings of First International Workshop on Crowd Simulation (V-CROWDS'05)*, Lausanne, Switzerland, November 24–25, 2005.

Yersin, B., Ma, J., Pettr, J. and and Thalmann, D. (2009). Crowd patches: populating large-scale virtual environments for real-time applications. *Proceedings of Proceedings of the 2009 symposium on Interactive 3D graphics and games*, Boston, Massachusetts, February 27–March 1, 2009, pp. 207–214.

Yu, C. and Peuquet, D. J. (2009). A GeoAgent-based framework for knowledge-oriented representation: Embracing social rules in GIS. *International Journal of Geographical Information Science* 23(7), pp. 923–960.

Zhang, T. and Tsou, M.-H. (2009). Developing a grid-enabled spatial Web portal for Internet GIServices and geospatial cyberinfrastructure. *International Journal of Geographic Information Science* 23(5), pp. 605–630.

Zhao, Y. and Murayama, Y. (2007). A new method to model neighborhood interaction in cellular automata-based urban geosimulation. In: Shi, Y., Albada, G. D. and Dongarra, J. (eds) *Lecture notes in computer science 4488: computational science (ICCS 2007)*. Berlin: Springer, pp. 550–557.