Agent-Based Models[1]

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The genealogy of agent-based models

Agent-based models (ABMs) are quite often confused with cellular automata (CA) both in methodology and application (Benenson and Torrens 2004; Torrens and O'Sullivan 2001). Both are, at their cores, approaches to model-building and simulation. They share a common ancestry in the mathematics for what would later become computer science (although their family trees are relatively disparate, as I will discuss shortly) and enjoy the spotlight in equal abundance in geocomputing and cyberinfrastructure research. They are quite often used to simulate the same or similar phenomena and systems in the geographical sciences. Moreover, the properties and parameterization of CA and ABMs are, to some extent, commutative. This explains the blurring of their distinctions in academic work at least partially. Yet, the two methodologies *are* fundamentally inconsonant and the differences between them are of profound significance to their suitability to and use in simulation of geographical phenomena.

The origins of CA are relatively transparent, as they share their foundation with that of all digital computers, in the 1930s following work by Turing on the computability of mathematics (Turing 1936, 1938) and in the 1940s following work by von Neumann and Ulam on the design of the first digital computing machines (von Neumann 1951, 1956; von Neumann and Burks 1966). The methodological foundation of CA as they are used today has changed little since that time. By contrast, the foundations of ABMs are relatively translucent. The methodology for ABMs was conceived alongside CA but has adapted and mutated significantly since the 1930s and 1940s, drawing particular inspiration from work in the late-1940s by Shannon on information theory (Shannon 1948) and by Weiner on cybernetics (Weiner 1948) as well as Turing's work in the 1950s on intelligent machines (Turing 1950). While CA might appropriately be considered as

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the grandmother of computer science, ABMs are, in some ways, broadly allied with information science, and specifically connected to artificial intelligence research.

Today, a lot of the research undertaken in CA methodology takes place under the moniker of computer science theory. Work on ABMs, however, is more broadly diffused through the computer and information sciences as well as electrical engineering: artificial neural networks; context-aware computing; bots, spiders and webcrawlers; expert systems; robotics; control systems; and so forth. It is relatively easy to say determine whether something is a cellular automaton and whether it is not. Determining when a model is agent-based or not is relatively difficult, largely because ABM research is more domain-specific than that of CA.

The distinction in CA and ABM genealogy is also evident in their use in the geographical sciences. CA were popularly introduced to geography through diffusion modeling, following Hägerstrand's research on innovation diffusion in the late-1960s (Hägerstrand 1967) and Tobler's work on urban growth modeling in the early-1970s (Tobler 1970). Agent-based modeling began to appear in the geography literature stealthily and at a much later time. The earliest geographical ABMs were developed by Itami and colleagues in the late-1980s to model recreation and wayfinding behavior along trails (Itami 1988). ABMs also trickled into geographic research through modeling work being done in the social sciences (Epstein 1999), game theory (Epstein and Axtell 1996), physics (Schweitzer 2003), biology (Deutsch and Dormann 2004), and entomology (Bonabeau et al. 1999) as part of an interdisciplinary focus on complex adaptive systems, as well as some pioneering and accessible work in computer graphics (Reynolds 1987).

How agent-based models (and their variants) work

ABMs are really quite simple in their basic design. The agents in ABMs are *processing devices* and in this sense they are computing media, with obvious advantages for their deployment as computable models in simulation. Agents are used to process different things, depending upon the applications that they are developed for. Robotic agents may be used to process chemical or electrical signals, for example. The agents in an ABM are almost always used to process data or information.

As the terminology suggests, ABMs are also *models*. They are most often used as in silico abstractions of something from the real world or something that exists in conceptual form. ABMs thus act as a surrogate representation of some phenomenon or system for the purposes of experimentation or scenario-building.

Moreover, ABMs are *agent-based*. They are a particular class of model used as proxies for agency or as vehicles to study agency. This is where their design and application can become intricate, largely because this property broadens the scope of agent-based modeling from computing to any discipline in which agency is important or even relevant. The list of related disciplines is long and includes geography, philosophy, political science, business studies, psychology, and organization theory, but is almost limitless. Even non-sentient material particles may be ascribed a level of agency.

There are many variants of ABM. Many (although not all) ABMs are designed as *agent automata*. Such ABMs are based on the same state–rule–input architecture that is fundamental to all automata, including CA, and could be represented as follows.

$$A \sim (S,R);\ S = \{S^1,S^2,\ldots,S^k\};\ R: (S_t,I_t)\rightarrow S_{t+1}$$

The A in the equation above represents an automaton with states that describe its attributes (perhaps in a particular place and/or time). The states may be derived from a discrete set, indexed from 1 to k, for example. The automaton's states are malleable to the influence of a rule (R) that could be used to govern transition among states from {S} as time changes from t → t+1, taking the state at time t and input (I) from another automaton or external stimulus into consideration. We can consider automaton A to be an agent automaton by simply characterizing states {S}, rules (R), and input (I) in terms of agency. We could, for example, index emotional states in {S}, where S^1 denotes happiness and S^2 denotes misery. The rule (R) in this exercise might be used to govern transition from a state of happiness to misery. We could formulate this rule as a set of if–then conditions: studying for a final exam in one of my courses might generate a level of misery, for example, but if you receive a lot of positive input (good lecture notes, a well-written text book), you will pass the exam and become quite happy (at least until your next exam!).

This is a very simple example, but interpretation of agency, states, and rules that govern state changes can be quite sophisticated. In human geography, we may interpret R in terms of choice,

decisions, trade-off, utilities, cognition, rationale, bias, and so forth. Within physical geography, R may be based on mechanisms, processes, events, conditions, laws and so forth. Input (I) is almost always determined geospatially or spatiotemporally (I_t) in geographic examples. The interplay between states, rules, input, and time offers the flexibility in a modeling methodology to cover *everything* of interest to a geographer. Agent automata are universal computers: anything that can be computed can be represented as or evaluated by an agent automaton. The methodology does not constrain the range of things that can be modeled; this is a characteristic that is unique among geographic modeling schemes.

Other classes of ABM exist and are flavored, for the most part, by their domain-specific origins and applications. *Individual-based models* are usually designed to represent the behavior of a single agent. This might include an institution, a mind, or a predatory animal.

Multi-agent models are generally built with many individual agents, each of which may play a different role or assume a set of distinct tasks in the model. Such models generally focus on the interactions between individuals or units, such as the development of spatial ontology through conversation or gene swapping along chromosomes through spatial interaction in genome-space.

Multi-agent systems adopt a synoptic view, often from the bottom-up, to consider individual agency in the context of a larger or collaborative phenomenon. A multi-agent system might be designed to explore dynamics in the economic geography of collaboration among car showrooms along a retail strip or the dynamics of conflict within an antisocial mob, for example. Often, the system that is considered is treated as a complex adaptive system and the interplay between agents and agent attributes is used to explore issues of emergence, feedback, self-organization, phase shift, and so on.

Intelligent agents are usually designed with a level of artificial intelligence or to explore the components of general or task-specific intellect. An intelligent agent model may be built to optimize a location-allocation scenario, for example, or to develop a winning space-time strategy in a virtual soccer game. ABMs of intelligence are generally built to represent spatial thinking, to explore human cognition as it relates to spatial way-finding, for example, or to automate the steering for a train in an intelligent transport system.

There is also a class of agents that might best be described as *bots*, owing to the resemblance that they share with robots (Latombe 1991; Leonard 1997; Pallman 1999). Such agent models are

designed to automate tasks, such as retrieving data for a Geographic Information System, or fanning out across a massive spatial network of client and server machines to map the World Wide Web.

Applications of agent-based models in the geographical sciences (and geographical applications of agent-based models in other sciences)

ABMs have been deployed for a wide variety of geographical applications, both within and outside the geographical sciences. Indeed, space and spatial thinking have emerged as a central concept in ABM modeling with the result that the methodology is, to some extent, catalyzing an infusion of geography into other disciplines, reversing the trend in quantitative geography for the field to poach methodology from other sciences.

Movement (the essence of geography) is one of the key distinguishing characteristics of ABMs. CA do not support movement of automata entities. Information can move in CA; state information may diffuse through CA (thereby giving the illusion of movement) (Faith 1998). The cells within a CA lattice are not generally designed to alter their position through locomotion, however. Agents in an ABM, on the other hand, may be designed with free motion as vectors in a field-space. Not surprisingly, ABMs have enjoyed relatively widespread use in the geographical sciences and elsewhere as models of movement.

ABMs are perhaps most widely used to study geography through local-scale movement, an application area that stems in large part from the popularity of ABM use in computer graphics for flocking and schooling behavior (Reynolds 1987, 1999). ABMs form the core of several models of vehicle traffic, for example. The standard routine for simulating vehicle trips is through the four-step travel model. The four steps in question are trip generation, trip distribution, mode choice, and trip assignment. ABMs can be used in any of these modules, but have also seen popular application as a fifth step in this scheme, traffic micro-simulation, in which the movement behaviors of cars along a section of road as part of a trip are simulated in varying degrees of detail. Nagel was among early adopters of ABMs for these purposes and developed a series of models designed to simulate driver and car behavior along roads (Nagel et al. 1996a; Nagel et al. 1996b; Nagel and Schrekenberg 1995). He was able to successfully generate new theories regarding the physics of traffic jams and congestion along roads as a result

of this modeling work. It is worth noting that while agent-based in design and conceptual foundation, these models were realized as CA, with "car-size" cells as the lowest resolution spatial unit. The algorithms in those models later formed the core of the TRANSIMS program to develop behavioral models to simulate the space-time activity of all cars on all roads over an entire city (Nagel et al. 1999).

Almost concurrently, researchers in zoology and entomology were developing ABMs to explore local movement in animal and insect populations. This area of research in zoology is encapsulated under the field of animats (Terzopoulous et al. 1994), which is focused on the development of models of animal behavior and collective action within systems. Animat research (Meyer and Guillot 1994) ranges from evolving and mutating creatures in silico (Meyer and Guillot 1994) to a fitness function driven by the subsequent ability of evolved animats to ambulate over land and under water (Sims 1994), to mimicking the movement and gait of animals for applications in robotics (Magnenat-Thalmann and Thalmann 1994). Geographic ABM work in entomology is focused on modeling the search and foraging behavior of flying and crawling insects, often considering their collective swarm intelligence (Bonabeau et al. 1999).

The earliest use of ABMs to model local-scale human movement was by Gipps and Marksjö in the mid-1980s (Gipps and Marksjö 1985). Their models, while agent-based, were actually built and run as CA. The models were designed to mimic the velocity, way-finding, and movement behavior of pedestrian agents in a "person-sized" cell. The presence of a person in a cell was encoded as a state variable and that information was diffused within a larger CA lattice subject to input regarding the presence of agents in neighboring cells and under the direction of rules of movement based navigation between an origin and a destination cell. While simple, the models generated realistic-looking dynamics mimicking the collective flow of pedestrians along sidewalks. CA models of agent-based design have been developed in much the same form to represent individual movement within large-scale crowds of shoppers in urban retail districts (Haklay et al. 2001) and in street carnivals (Batty et al. 2003). These models are used to explore issues of infrastructure placement and urban design for the most part. A group of physicists have also developed true ABMs of human movement, treating mobile pedestrians as agent vectors in a field-space. Work of this kind in physics begins with an assumption that the density of agents is such that the dynamics of a crowd of agents will succumb to characteristics observed in particle movement and granular flow (Helbing et al. 2001). The models then proceed to ascribe

movement behaviors to agents that mimic the individual trajectories of particles in physical systems. Helbing and Vicsek are among early pioneers of such work (Helbing et al. 2001; Vicsek 2003). Helbing has adapted his physics-based models to account for social forces of repulsion and attraction in crowds that may better explain movement behavior (Helbing and Molnár 1995).

ABMs have also been used to explore geography under meso- and macro-scale movement. Anthropologists and archaeologists, in particular, have developed models of hunter-gatherer movement in efforts to "hindcast" the migration patterns of groups in the past and in some instances have been used to target archaeological excavation work in landscapes (Lake 2000). Often, such models are linked to simulations of land-use and land cover change dynamics as well as past predictions of climate and vegetation shifts. They may be used to explore and cross-relate different hypotheses regarding the formation of cultural groups, agricultural practices, or nomadic strategies. Geographers have also used agent-based models of meso-scale movement to explore the relationship between shifting patterns of migration and spatiotemporal agglomeration and the formation of urban settlements (Batty 2001). This line of research inquiry has also attracted the interest of physicists because of its relevance to theories and methodologies for explaining and modeling random walks and Brownian motion (Schweitzer 2003). ABMs are also popularly used in political science to look at the emergence of cooperation and conflict. Some aspects of this work have been adopted by geography researchers in the land-use and land cover change community and allied to meso-scale movement for the purposes of examining shifting cultivation patterns and deforestation in the context of resource use and land markets (Parker et al. 2003). There is also a cohort of geographers using ABMs to model meso-scale movement of families through the urban landscape and such models are most often used to simulate housing search and residential choice across the lifecycle of families (Benenson 1998, 1999; Benenson et al. 2002; Portugali 2000; Portugali et al. 1994).

Models of urbanization and urban growth are particularly well-suited to agent-based methodologies and ABMs have begun to be used this context. Urbanization models have been developed with agents as settlers, developers, and relocating households faced with location choices amid economic, physical, social, cultural, and environmental geographies. The SIMPOP models developed by Sanders and colleagues are an early example of ABM use in this fashion, whereby individual towns and cities are treated as competing and collaborating agents amid a larger metropolitan or megalopolitan urban system (Sanders et al. 1997). CA-based models of

urbanization and urban growth have also been built with agent-based functionality, largely to simulate the dynamics of land-use transition in city-systems or to evaluate the role of complex adaptive human, social, and economic dynamics in shaping the rate and pattern of urbanization under varying scenarios (Clarke et al. 1997; Engelen et al. 2002; Semboloni 1997; Semboloni et al. 2004; Torrens 2002, 2003, 2006; Webster and Wu 1998; White and Engelen 1997; Wu 1998, 2002; Wu and Webster 2000). This latter focus is an area of research that is particularly active in the geographical sciences.

ABMs of social geography date to the mid-1970s, following popular work by Schelling and Sakoda with grid-based conceptual models of socio-spatial segregation dynamics (Sakoda 1971; Schelling 1969, 1971, 1978). These models, while not originally computer-based and actually specified as CA, are significant in the social sciences as being exemplars of the utility of using simple rules among many interacting agents as a vehicle for exploring socio-spatial dynamics. Both Schelling and Sakoda developed very simple models of segregation between binary groups over space and evaluated the conditions under which populations could be polarized following biases in their feelings toward groups of an opposite kind, and the tipping-point (what is known as a phase shift in the complexity literature) at which socio-spatial polarization would occur. Indeed, Schelling shared the Nobel Prize in economics in 2005 for related work on the dynamics of conflict and cooperation. More recent work in this area by geographers has focused on modeling micro-scale segregation dynamics in residential choice (Benenson and Omer 2003; Benenson et al. 2002) and examination of the computational geometry of agents' neighborhood filters for satisfaction with their residential choices (Shi and Pang 2000).

ABMs have also seen use for geographical research as a methodology for simulating risk and vulnerability. Emergency and catastrophic events, by their very nature, are all but impossible to explore and experiment with in the real world. Models are therefore an invaluable tool in evaluating plans and scenarios for vulnerability and resilience. Such models need to be realistic to be useful, however, and ABM methodologies can play an important role in modeling the human agency in panic, evacuation, response, and so forth. Vicsek and Farkas have published some of the seminal ABM work in modeling escape panic, using physics to explore bottlenecks at exits and the dynamic behavior of crowds as excitable media (Farkas et al. 2002; Vicsek 2003). Helbing and colleagues have modeled the success of varying strategies for escape from

buildings in emergency situations, from the perspective of individual and collective human behavior (Helbing et al. 2000).

The future of agent-based models and modeling in the geographical sciences

The use of ABMs in the geographical sciences, while growing in popularity, is still in a stage of relative infancy as a topic of academic inquiry. Some relatively serious challenges remain in advancing the research agenda further.

Much development work with ABMs in the geographical sciences is focused on the development of methods and tools. Relatively little work is performed with theory-building as a primary objective. This is understandable given the relative nascence of the methodology. There is a more long-standing problem, however, with the relationship between ABMs and theory. Much geographical theory abstracts from the micro-scale. Essentially put, we lack the theory to derive rules for ABMs that treat individuals and scale all the way up to large macro-systems.

Geographers encounter the same problems with data and dataware to support ABMs. Detailed data-sets that might be used to parameterize and calibrate fine-scaled ABMs are rarely available, owing to privacy concerns of sharing data about individuals, the difficulties of collecting data longitudinally over long time periods, and the qualitative training of generations of geographers used to thinking spatially in aggregate terms. This is beginning to change; individual-scale data are routinely collected in transport studies, for example, and geospatial technologies for automated tracking and sensor webs do provide near-real-time data feeds for whole study groups in some instances. Geographers and other scientists are also turning to innovative methods for generating “synthetic data populations” for use in ABMs.

Validating ABMs is hugely problematic because quantitative geography has not focused on development of techniques for measuring and analyzing individual units as part of massively interactive, dynamic, and non-linear systems. This has led to some criticism of ABMs as “toy models” in the absence of robust quantitative schemes that would allow for agent-based simulations to be registered to or compared with real-world systems.

ABMs are often allied with complex adaptive systems, yet research into spatial complexity has gained momentum only in recent years. This has led to difficulties in determining the signatures of complexity in ABMs and the real-world phenomena that they are used to simulate. In

particular, it has been quite difficult for developers of ABMs to create a methodological structure for capturing novel spatial ensembles and facilitating adaptation and learning in their simulations.

Many ABM developers in the geographical sciences have turned to Geographic Information Science in an attempt to overcome some of these difficulties, with some success thus far. The coupling of ABMs with Geographic Information Systems has provided a framework for structuring the design and application of ABMs for theory-experimentation, as well as providing a natural platform for integrating and reconciling diverse data-sets and validation schemes.

Further reading

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